

## Mathematische Methoden

3. *Exercise* to be presented on 10.1.2012 WS 2011/12

26. Compute the following definite integral:

$$\int_1^6 x^2 e^{-\frac{3}{4}x} dx$$

27. Compute the following definite integral:

$$\int_1^e \frac{2 \ln x^2}{x} dx$$

28. Compute the following indefinite integral for some constant  $a \neq 0$ :

$$\int -ax e^{-ax^2} dx$$

29. Compute the following definite integral:

$$\int_1^8 \frac{1}{3 + \sqrt{x+8}} dx$$

30. Compute the following indefinite integral by partial fractions:

$$\int \frac{3x^4 - 5x^3 - 16x^2 + 8x + 27}{x^2 - x - 6} dx$$

31. Compute the following indefinite integral by partial fractions:

$$\int \frac{4x^5 + 6x^3 + 2x - 2}{x^3 + x} dx$$

32. Given is the following function  $H(z)$ :

$$H(z) = \int_2^{3z} 4y^2 z - e^{3yz^2} dy$$

Determine the first derivative  $H'(z)$  in two ways, i.e. by the Leibniz Theorem and by direct integration.

33. Solve the following differential equation:

$$\frac{1}{2} \dot{x}(t) = e^{-3x(t)}$$

34. Solve the following differential equation with the initial condition  $x(0) = 3$ .

$$\dot{x}(t) = t(x - 1)^2$$

35. Solve the following differential equation with the initial condition  $x(0) = 0$ .

$$e^{2t} \dot{x}(t) - x^2(t) - 2x(t) = 1$$

36. Solve the following differential equation:

$$\dot{x}(t) = 2t x(t) + e^{t^2}$$

37. Solve the following differential equation for  $t > 1$ :

$$\dot{x}(t) - t = \frac{t}{t^2 - 1} x(t)$$